

# Structure Formation in Modular Entropic Gravity: The MEG-Native Picture

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## Abstract

We show that the structure formation problem identified in earlier exploratory work — in which a temporal screening transition from  $\mu_{\text{prim}} \approx 5.4$  to  $\mu_{\text{gal}} \approx 0.06$  was found to collapse the gravitational potential — does not arise in MEG when the actual screening law is applied correctly. MEG’s screening is environmental, not temporal: the one-loop-derived screening law  $\ell_{\text{env}}(\rho_b) = \ell_0[1 + (\rho_b/\rho_0)^{1/3}]^{-1/4}$  activates only where the *local* baryon density exceeds the universal screening density  $\rho_0 \approx 10^{-21} \text{ kg/m}^3$ . On linear cosmological scales, the mean baryon density satisfies  $\bar{\rho}_b/\rho_0 \sim 10^{-6}$  today and  $\sim 10^{-3}$  at the onset of structure formation. The vacuum entropy response is therefore completely unscreened on all linear cosmological scales at all post-recombination epochs.

Consequently, the modified Poisson equation retains the full primordial coupling  $\mu_{\text{prim}} \approx 5.4$  throughout the linear growth epoch. Baryons grow with effective matter density  $\Omega_m^{\text{eff}} = \Omega_b(1 + \mu_{\text{prim}}) \approx 0.32$ , reproducing the  $\Lambda$ CDM growth factor  $D(a) \propto a$  during matter domination. The gravitational potential persists at  $\sim 80\%$  of its recombination value through to the present epoch, with the  $\sim 20\%$  loss attributable to standard  $\Lambda$ -induced decay.

Screening activates only inside nonlinear structures — virialised halos, galaxy cores, the Solar System — where  $\rho_b \gg \rho_0$ . This is the regime where MEG’s rotation curve phenomenology operates. The cosmological (linear) and galactic (nonlinear) regimes are governed by the same screening law but evaluated at vastly different densities, producing vastly different effective couplings:  $\mu_{\text{eff}} \approx \mu_{\text{prim}} \approx 5.4$  on linear scales, and  $\mu_{\text{eff}} \approx \mu_{\text{gal}} \approx 0.06$  inside galaxies.

The Helmholtz Jeans scale identified in the CMB paper (P45) at  $k_\mu \approx 0.08 \text{ Mpc}^{-1}$  at recombination moves to  $k_\mu \gtrsim 8 \text{ Mpc}^{-1}$  by  $z = 10$ , placing all structure-formation modes ( $k \lesssim 1 \text{ Mpc}^{-1}$ ) deeply in the sub-Helmholtz (pressureless) regime throughout the growth epoch. There is no Jeans suppression during structure formation.

Under the adiabatic condition (both components seeded by the same inflationary perturbation), MEG’s linear structure formation matches  $\Lambda$ CDM, with departures appearing only at nonlinear/galactic scales where the environmental screening activates — which is precisely the regime where MEG’s distinctive rotation curve predictions operate.

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# 1 Introduction

An earlier exploratory analysis investigated whether MEG could sustain the gravitational potential through the matter-dominated epoch and support baryon clustering. The analysis modelled the screening transition as a global, time-dependent function — a tanh step from  $\mu_{\text{prim}} \approx 5.4$  to  $\mu_{\text{gal}} \approx 0.06$  at some transition redshift — and found that any such transition collapses the potential by an order of magnitude.

This paper shows that the analysis was based on a false premise. MEG’s screening is not a global temporal transition. It is a *local, density-dependent* environmental effect, derived from first principles in the monograph through a one-loop QFT calculation in a compressible baryonic medium. The screening law depends on the *local* baryon density  $\rho_b$ , not on the scale factor  $a$  or the redshift  $z$ .

When the actual screening law is applied, the structure formation problem dissolves: the vacuum entropy response is unscreened on all linear cosmological scales, the enhanced gravitational coupling persists throughout the growth epoch, and baryons grow exactly as in  $\Lambda$ CDM.

## 2 The MEG Screening Law

### 2.1 The one-loop derivation

The monograph derives the environmental screening law from a one-loop QFT calculation. In a compressible baryonic medium at density  $\rho_b$ , integrating out density fluctuations produces a non-analytic  $|k|$  term in the static self-energy of the vacuum entropy field  $S$  (the Lindhard cusp). After Gaussian coarse-graining and minimisation of the variance ratio, the result is:

$$\ell_{\text{env}}(\rho_b) = \ell_0 \left[ 1 + \left( \frac{\rho_b}{\rho_0} \right)^{1/3} \right]^{-1/4}, \quad (1)$$

where  $\ell_0 \approx 11$  kpc is the vacuum coherence length and  $\rho_0$  is the universal screening density:

$$\rho_0 = \frac{c^2}{8\pi G} \frac{\zeta_{\text{SM}}}{\ell_0^2} \approx 10^{-21} \text{ kg/m}^3. \quad (2)$$

### 2.2 Key properties

The screening law (1) has the following properties:

- **Argument:** The *local* baryon density  $\rho_b$  — not the scale factor  $a$ , not the redshift  $z$ , not a phenomenological transition function.
- **Vacuum limit:**  $\rho_b \rightarrow 0 \Rightarrow \ell_{\text{env}} \rightarrow \ell_0$  (unscreened).
- **High-density limit:**  $\rho_b \gg \rho_0 \Rightarrow \ell_{\text{env}} \propto \rho_b^{-1/12}$  (strong screening).
- **Onset:** Screening becomes significant when  $\rho_b \gtrsim \rho_0 \approx 10^{-21} \text{ kg/m}^3$ .

### 2.3 The effective coupling

The MEG-modified Poisson equation is:

$$k^2 \Phi = 4\pi G a^2 [1 + \mu_{\text{eff}}(k, \rho_b)] \rho_b \delta_b, \quad (3)$$

where  $\mu_{\text{eff}}$  is determined by the ratio of the screening length to the coherence length. In the unscreened vacuum ( $\rho_b \ll \rho_0$ ),  $\mu_{\text{eff}} = \mu_{\text{prim}} \approx 5.4$ . In strongly screened environments ( $\rho_b \gg \rho_0$ ),  $\mu_{\text{eff}} \rightarrow \mu_{\text{gal}} \approx 0.06$ .

The key point:  $\mu_{\text{eff}}$  depends on  $\rho_b$ , not on  $a$ . The same screening law produces both the cosmological coupling and the galactic coupling, evaluated at different densities.

## 3 Why Screening Does Not Operate on Linear Scales

### 3.1 The density hierarchy

On linear cosmological scales, the baryon density is the cosmic mean:

$$\bar{\rho}_b(a) = \rho_{b,0} a^{-3}, \quad \rho_{b,0} = \frac{3H_0^2 \Omega_b}{8\pi G} \approx 4.2 \times 10^{-28} \text{ kg/m}^3. \quad (4)$$

The ratio to the screening density:

$$\frac{\bar{\rho}_b(a)}{\rho_0} = \frac{\rho_{b,0}}{\rho_0} a^{-3} \approx 4.2 \times 10^{-7} a^{-3}. \quad (5)$$

$z$	$a$	$\bar{\rho}_b/\rho_0$	Screening status
1090 (rec)	$9.2 \times 10^{-4}$	$5.4 \times 10^2$	screened <sup>†</sup>
200	$5.0 \times 10^{-3}$	3.4	marginal
133	$7.5 \times 10^{-3}$	1.0	threshold crossing
100	$9.9 \times 10^{-3}$	$4.3 \times 10^{-1}$	marginal
30	$3.2 \times 10^{-2}$	$1.3 \times 10^{-2}$	unscreened
10	$9.1 \times 10^{-2}$	$5.6 \times 10^{-4}$	unscreened
3	$2.5 \times 10^{-1}$	$2.7 \times 10^{-5}$	unscreened
0	1	$4.2 \times 10^{-7}$	unscreened

<sup>†</sup> See Remark 1.

The mean baryon density crosses below the screening threshold  $\rho_0$  at  $z \approx 133$ . Throughout the entire structure formation epoch ( $z \lesssim 30$ ), the mean baryon density is at least two orders of magnitude below the screening density.

**Remark 1** (Screening at recombination). *At recombination ( $z \approx 1090$ ),  $\bar{\rho}_b/\rho_0 \approx 540$ , giving  $\ell_{\text{env}} \approx 0.57 \ell_0$ . The environmental screening is therefore active at the mean density at recombination.*

*This does not affect the primordial kernel  $\mu_{\text{prim}}$  used in P45. The primordial kernel is the frozen vacuum memory from inflation — the modular entropy perturbation imprinted during inflation and frozen by Hubble friction (the persistence regime of P45, where  $\omega_\mu \ll H$ ). This component is not generated by the recombination-era baryon density and is not subject to environmental screening by it. The environmental screening law governs the local vacuum response to present baryon density (the galactic component  $\mu_{\text{gal}}$ ), not the primordial vacuum memory.*

*The two-component kernel of P45 separates these:  $\mu_{\text{prim}}$  (primordial, from inflation, unscreened) and  $\mu_{\text{gal}}$  (environmental, from local density, screened). The table above applies to the environmental component.*

### 3.2 The unscreened effective coupling

For the structure formation epoch ( $z \lesssim 30$ ), evaluating the screening law (1) at the cosmological mean density:

$$\ell_{\text{env}}(\bar{\rho}_b) = \ell_0 \left[ 1 + \left( \frac{\bar{\rho}_b}{\rho_0} \right)^{1/3} \right]^{-1/4} \approx \ell_0 [1 + \mathcal{O}(10^{-2})]^{-1/4} \approx \ell_0. \quad (6)$$

The correction to  $\ell_{\text{env}}$  from environmental screening is a few percent or smaller for  $z \lesssim 30$  on linear scales (e.g.  $\ell_{\text{env}}/\ell_0 \approx 0.95$  at  $z = 30$ ,  $\approx 0.98$  at  $z = 10$ ). The effective coupling is therefore:

$$\boxed{\mu_{\text{eff}}(\text{linear scales, } z \lesssim 100) \approx \mu_{\text{prim}} \approx 5.4.} \quad (7)$$

**Result.** The vacuum entropy response is completely unscreened on linear cosmological scales throughout the structure formation epoch ( $z \lesssim 30$ ). The enhanced gravitational coupling  $\mu_{\text{prim}} \approx 5.4$  persists unscreened. The environmental screening threshold is crossed at  $z \approx 133$ ; above this redshift, the environmental response is partially screened, but this affects only the environmental component, not the primordial vacuum memory (Remark 1).

### 3.3 Screening inside nonlinear structures

By contrast, inside virialised structures:

- Galaxy cores:  $\rho_b \sim 10^{-22}$  to  $10^{-20}$  kg/m<sup>3</sup>, giving  $\rho_b/\rho_0 \sim 0.1$  to  $10$ .  $\ell_{\text{env}} \approx 0.7\ell_0$  to  $0.6\ell_0$ .  $\mu_{\text{eff}}$  significantly reduced.
- Solar System:  $\rho_b \sim 10^{-18}$  kg/m<sup>3</sup> (Solar neighbourhood), giving  $\rho_b/\rho_0 \sim 10^3$ .  $\ell_{\text{env}} \approx 0.2\ell_0$ .  $\mu_{\text{eff}} \approx \mu_{\text{gal}} \approx 0.06$ . GR recovered.

The two regimes — cosmological (unscreened,  $\mu \approx 5.4$ ) and galactic (screened,  $\mu \approx 0.06$ ) — are not two different physical mechanisms. They are the *same* screening law evaluated at different densities.

## 4 The Persistent Baryon-Coupled Enhancement

The modified Poisson equation  $(1 + \mu_{\text{prim}})\rho_b\delta_b$  is used throughout this paper. This section derives it from MEG’s primordial entropy component, rather than assuming it.

### 4.1 The gravitationally relevant variable

The primordial MEG component should not be treated as an independent clustering field amplitude. The microscopic entropy perturbation  $S$  oscillates rapidly ( $\omega_\mu \gg H$ , the oscillating mode mechanism of P45), but the gravitationally relevant quantity is the coarse-grained oscillation energy density:

$$\rho_\chi \equiv \langle \rho S \rangle, \quad \delta_\chi \equiv \frac{\delta \rho_\chi}{\bar{\rho}_\chi}. \quad (8)$$

For sub-Helmholtz modes after recombination (which includes all structure-formation modes, as shown in Section 6), this component has  $\langle w \rangle \simeq 0$ , so:

$$\bar{\rho}_\chi \propto a^{-3}. \quad (9)$$

### 4.2 The adiabatic condition

Both the baryon density perturbation and the primordial entropy memory are sourced by the same inflationary curvature perturbation  $\mathcal{R}$ . In P46, the modular entropy amplitude  $A_0$  was shown to be proportional to  $\sqrt{A_s}$  through the causal-ball modular projection. The entropy energy density perturbation is therefore sourced by the same  $\mathcal{R}$  that sources  $\delta_b$ .

On the linear growing mode, this gives the adiabatic condition:

$$\delta_\chi = \delta_b \quad (\text{linear adiabatic mode}). \quad (10)$$

This is not an assumption about the entropy field “clustering like CDM.” It is a consequence of both components being seeded by the same inflationary perturbation — the standard adiabatic initial condition that inflation produces.

**Remark 2** (Adiabatic vs isocurvature). *The derivation requires the primordial entropy component to be adiabatic, not isocurvature. This is the standard inflationary prediction: single-field slow-roll inflation produces adiabatic perturbations in all components sourced by the same inflaton. Observational constraints from the CMB (Planck) strongly limit isocurvature contributions. The adiabatic condition is therefore both theoretically well-motivated and observationally supported.*

### 4.3 The enhanced Poisson equation

With  $\delta_\chi = \delta_b$ , the Poisson equation becomes:

$$\begin{aligned} k^2 \Phi &= 4\pi G a^2 (\bar{\rho}_b \delta_b + \bar{\rho}_\chi \delta_\chi) \\ &= 4\pi G a^2 \bar{\rho}_b \delta_b \left( 1 + \frac{\bar{\rho}_\chi}{\bar{\rho}_b} \right). \end{aligned} \quad (11)$$

Defining

$$\mu_{\text{prim}} \equiv \frac{\bar{\rho}_\chi}{\bar{\rho}_b}, \quad (12)$$

we obtain:

$$k^2 \Phi = 4\pi G a^2 (1 + \mu_{\text{prim}}) \bar{\rho}_b \delta_b. \quad (13)$$

Because both  $\bar{\rho}_\chi$  and  $\bar{\rho}_b$  scale as  $a^{-3}$ , the ratio  $\mu_{\text{prim}}$  is *constant* during the linear growth epoch. Environmental screening does not erase this term on linear cosmological scales, because  $\bar{\rho}_b \ll \rho_0$  after  $z \sim 133$  (Section 3).

**Result.** The persistent MEG-enhanced Poisson equation (13) is derived, not assumed. It follows from three established results:

1. The oscillating entropy modes have  $\langle w \rangle \simeq 0$  and  $\bar{\rho}_\chi \propto a^{-3}$  (P45).
2. The entropy energy density and baryons are seeded by the same inflationary curvature perturbation, giving  $\delta_\chi = \delta_b$  (adiabatic condition, P46).
3. Environmental screening is negligible on linear scales (Section 3).

MEG does not need an independently clustering entropy fluid. It needs a persistent primordial entropy-energy component whose density contrast is adiabatically locked to the baryon perturbation on linear scales.

## 5 The Growth Equation on Linear Scales

### 5.1 The MEG growth equation

From the derived Poisson equation (13):

$$k^2 \Phi = 4\pi G a^2 (1 + \mu_{\text{prim}}) \rho_b \delta_b = 4\pi G a^2 \rho_m^{\text{eff}} \delta_b, \quad (14)$$

where  $\rho_m^{\text{eff}} = (1 + \mu_{\text{prim}}) \rho_b$ .

The baryon growth equation in the matter-dominated era is:

$$\delta_b'' + \left( 2 + \frac{d \ln H}{d \ln a} \right) \delta_b' = \frac{3}{2} \Omega_b(a) (1 + \mu_{\text{prim}}) \delta_b, \quad (15)$$

where primes denote  $d/d \ln a$ .

## 5.2 The background expansion

The background Friedmann equation uses:

$$H^2 = H_0^2 \left[ \Omega_m^{\text{eff}} a^{-3} + \Omega_\Lambda \right], \quad \Omega_m^{\text{eff}} = \Omega_b(1 + \mu_{\text{prim}}) \approx 0.32. \quad (16)$$

This is identical to  $\Lambda$ CDM with  $\Omega_m = 0.32$ , because the oscillating entropy modes contribute to the background energy density with  $\langle w \rangle \approx 0$  on all sub-Helmholtz scales (P45).

## 5.3 The growth factor

**Proposition 1** (Linear growth in MEG). *During matter domination, with  $\mu_{\text{eff}} = \mu_{\text{prim}}$  on linear scales and  $\Omega_m^{\text{eff}} = \Omega_b(1 + \mu_{\text{prim}})$  in the background, the linear growth equation reduces to the standard  $\Lambda$ CDM form with  $\Omega_m = \Omega_m^{\text{eff}}$ . The growing-mode solution is  $\delta_b \propto a$  during matter domination, and the growth factor from recombination to today is  $D(a=1)/D(a_{\text{rec}}) \approx 860$ .*

*Sketch.* Writing  $\delta_b \propto a^p$  in matter domination ( $\Omega_b(a)(1 + \mu_{\text{prim}}) = \Omega_m^{\text{eff}} = 1$  in matter domination), the growth equation gives  $p^2 + \frac{1}{2}p - \frac{3}{2} = 0$ , with the growing-mode solution  $p = 1$ .  $\square$

## 5.4 The gravitational potential

The gravitational potential evolves as:

$$\Phi(k, a) \propto \frac{(1 + \mu_{\text{prim}}) \delta_b(a)}{a} = \frac{(1 + \mu_{\text{prim}}) D(a)}{a}. \quad (17)$$

During matter domination,  $D(a) \propto a$ , so  $\Phi = \text{const}$  — the potential is constant. At late times,  $\Lambda$  causes the standard  $\sim 20\%$  potential decay, giving:

$$\frac{\Phi(a=1)}{\Phi(a_{\text{rec}})} \approx 0.79, \quad (18)$$

identical to  $\Lambda$ CDM under the adiabatic persistent-kernel assumption.

**Result.** Under the adiabatic condition ( $\delta_\chi = \delta_b$ , derived in Section 4), MEG's linear structure formation matches  $\Lambda$ CDM. The growth factor, the potential persistence, and the baryon power spectrum on linear scales all match  $\Lambda$ CDM, because the enhanced gravitational coupling  $\mu_{\text{prim}}$  is unscreened on these scales and the background expansion uses the same  $\Omega_m^{\text{eff}}$ .

# 6 The Helmholtz Jeans Scale During Structure Formation

## 6.1 The evolving Jeans scale

P45 identified a Helmholtz Jeans scale at  $k_\mu \approx 0.08 \text{ Mpc}^{-1}$  at recombination, where the effective equation of state transitions from  $w \approx 0$  (sub-Helmholtz, pressureless) to  $w \rightarrow 1/3$  (super-Helmholtz, radiation-like). This is a real effect at the CMB epoch and produces the testable prediction identified in P45.

However, the Helmholtz scale in comoving coordinates evolves with the scale factor:

$$k_\mu(a) = \mu a = \frac{a}{\ell_0}. \quad (19)$$

This is because  $\mu = \ell_0^{-1}$  is a *physical* wavenumber, and the comoving equivalent is  $k_\mu^{\text{com}} = \mu a$ .

$z$	$a$	$k_\mu$ [Mpc $^{-1}$ ]	Structure-formation scales
1090 (rec)	$9.2 \times 10^{-4}$	0.08	partially affected
100	$9.9 \times 10^{-3}$	0.90	all sub-Helmholtz
30	$3.2 \times 10^{-2}$	2.9	all sub-Helmholtz
10	$9.1 \times 10^{-2}$	8.3	deeply sub-Helmholtz
3	$2.5 \times 10^{-1}$	22.7	deeply sub-Helmholtz
0	1	90.9	deeply sub-Helmholtz

## 6.2 Consequence: no Jeans suppression during structure formation

All structure-formation modes ( $k \lesssim 1$  Mpc $^{-1}$ ) are deeply sub-Helmholtz during the entire growth epoch ( $z \lesssim 30$ ). Their effective equation of state is:

$$\langle w(k) \rangle = \frac{k_{\text{phys}}^2}{3(k_{\text{phys}}^2 + \mu^2)} \approx \frac{(k/a)^2}{3\mu^2} \ll 1 \quad \text{for } k \ll k_\mu(a). \quad (20)$$

For example, at  $z = 10$  with  $k = 0.1$  Mpc $^{-1}$ :  $k/k_\mu = 0.1/8.3 = 0.012$ , giving  $\langle w \rangle \approx 5 \times 10^{-5}$ . The entropy modes are pressureless on all relevant scales.

**Result.** The Helmholtz Jeans suppression identified in P45 is a *recombination-epoch* effect. During the structure formation epoch, the comoving Jeans scale has moved far above all structure-formation modes. There is no Jeans suppression of structure formation in MEG.

**Remark 3** (Consistency with P45). *This does not contradict P45. P45 correctly identifies  $k_\mu \approx 0.08$  Mpc $^{-1}$  at recombination as a testable CMB prediction. The present result adds that this scale evolves with  $a$ , so the prediction applies to the CMB but not to late-time structure formation. Both results follow from the same physics.*

## 7 Correction of the Temporal Screening Analysis

An earlier exploratory analysis modelled MEG screening as a global temporal transition and found catastrophic potential collapse. That analysis made two errors that are important to document explicitly.

### 7.1 Error 1: Temporal screening instead of environmental screening

The analysis modelled screening as a global, time-dependent transition:

$$\mu_{\text{eff}}(a) = \mu_{\text{prim}} \cdot \frac{1}{2} \left( 1 - \tanh \frac{a - a_{\text{scr}}}{\Delta a} \right) + \mu_{\text{gal}} \cdot \frac{1}{2} \left( 1 + \tanh \frac{a - a_{\text{scr}}}{\Delta a} \right). \quad (21)$$

This assumes that the entire universe transitions from  $\mu_{\text{prim}}$  to  $\mu_{\text{gal}}$  at a characteristic redshift  $z_{\text{scr}}$ . But MEG's actual screening law is:

$$\ell_{\text{env}}(\rho_b) = \ell_0 \left[ 1 + \left( \frac{\rho_b}{\rho_0} \right)^{1/3} \right]^{-1/4}, \quad (22)$$

which depends on the *local* baryon density, not on the epoch. On linear scales,  $\rho_b \approx \bar{\rho}_b \ll \rho_0$ , and the screening is negligible. The temporal transition is a caricature that does not represent MEG's physics.



This error is analogous to treating chameleon screening in  $f(R)$  gravity as a global phase transition rather than an environmental effect. The literature on screened modified gravity has been careful to distinguish between background cosmological evolution (which uses the unscreened theory) and local environmental effects (which use the screened theory). MEG's screening law has the same structure.

## 7.2 Error 2: Evaluating the Jeans scale at a fixed epoch

The analysis used the effective sound speed  $c_{\text{eff}}^2 = (k/a\mu)^2/[4 + (k/a\mu)^2]$  evaluated at recombination or at a fixed epoch, and found Jeans suppression for  $k \gtrsim 0.05 \text{ Mpc}^{-1}$ . This is correct at recombination, but the Jeans scale evolves as  $k_\mu = \mu a$ : as the universe expands, the comoving Jeans scale moves to higher  $k$ . During structure formation ( $z \lesssim 30$ ), all cosmologically relevant modes ( $k \lesssim 1 \text{ Mpc}^{-1}$ ) are deeply sub-Helmholtz and experience no Jeans suppression.

## 7.3 What the earlier analysis established

Despite the incorrect premises, the analysis established several results that remain valid:

- The correct empirical standard: MEG must reproduce  $\Phi(k, a)$  and  $\delta_b(k, a)$ , not literal  $\Lambda$ CDM particle clustering.
- The continuity condition: if a temporal screening transition *were* to occur, it would need to be extremely gradual. This is correct as a conditional statement, but the condition is not triggered because no temporal transition occurs.
- The large-scale growth result: with constant  $\mu_{\text{prim}}$ , the growth factor is  $D(a) \propto a$ . This is correct and is the actual MEG prediction.
- The energy-density reformulation: the clustering variable is  $\delta\rho_S/\bar{\rho}_S$ , not the field amplitude  $\delta S$ . This is physically correct, though it is not needed for the structure formation result once environmental screening is applied correctly.

# 8 The Complete MEG Structure Formation Picture

## 8.1 The derivation chain

1. **Background expansion:** The oscillating entropy modes contribute to the background energy density with  $\langle w \rangle \approx 0$ , giving  $\Omega_m^{\text{eff}} = \Omega_b(1 + \mu_{\text{prim}}) \approx 0.32$ . The Friedmann equation is identical to  $\Lambda$ CDM. (*Established in P45.*)
2. **Recombination:** The primordial kernel  $\mu_{\text{prim}} \approx 5.4$  gives the correct matter-radiation equality epoch  $z_{\text{eq}} \approx 3400$  and the correct CMB acoustic peak structure. (*Established in P45.*)
3. **Post-recombination growth:** On linear scales,  $\bar{\rho}_b \ll \rho_0$ , so the screening law gives  $\ell_{\text{env}} \approx \ell_0$  and  $\mu_{\text{eff}} \approx \mu_{\text{prim}}$ . Baryons grow with the standard  $\Lambda$ CDM growth factor  $D(a) \propto a$  during matter domination. (*Established in this paper.*)
4. **Potential persistence:**  $\Phi(a) \propto (1 + \mu_{\text{prim}})D(a)/a = \text{const}$  during matter domination, with standard  $\Lambda$ -induced decay at late times.  $\Phi(a=1)/\Phi(a_{\text{rec}}) \approx 0.79$ . (*Established in this paper.*)
5. **Jeans scale:** The Helmholtz Jeans scale  $k_\mu = \mu a = a/\ell_0$  evolves to  $k_\mu \gtrsim 8 \text{ Mpc}^{-1}$  during the growth epoch. All structure-formation modes are sub-Helmholtz (pressureless). No Jeans suppression. (*Established in this paper.*)

6. **Nonlinear regime:** When  $\delta_b$  becomes  $\mathcal{O}(1)$  and structures virialise, the local baryon density rises above  $\rho_0$  and screening activates. Inside galaxies,  $\mu_{\text{eff}} \rightarrow \mu_{\text{gal}} \approx 0.06$ . This is the regime where MEG’s rotation curve phenomenology operates. (*Established in the monograph.*)

## 8.2 The physical picture

### The MEG structure formation picture.

Baryons fall into their own enhanced gravitational field. The vacuum entropy field does not cluster separately — it enhances the gravitational response to baryon density perturbations through the modified Poisson equation with coupling  $\mu_{\text{prim}} \approx 5.4$ .

On linear scales, the mean baryon density is far below the screening threshold, so the enhanced coupling persists throughout the growth epoch. The growth factor, the potential history, and the baryon power spectrum all match  $\Lambda\text{CDM}$ .

Screening activates only inside nonlinear structures, where the local baryon density exceeds  $\rho_0$ . This produces the galactic-scale phenomenology (rotation curves, lensing) without affecting the cosmological linear growth.

The two regimes — cosmological (enhanced, unscreened) and galactic (screened) — are not separate mechanisms. They are the same screening law evaluated at different densities.

## 8.3 Comparison with $\Lambda\text{CDM}$

Observable	$\Lambda\text{CDM}$	MEG
$z_{\text{eq}}$	3400 (from $\Omega_{\text{cdm}}$ )	3400 (from $\mu_{\text{prim}}$ )
CMB peaks	matched	matched (P45)
$D(a) \propto a$	from CDM clustering	from enhanced baryon coupling
$\Phi(a=1)/\Phi_{\text{rec}}$	$\approx 0.79$	$\approx 0.79$
$\sigma_8$	from CDM power spectrum	from enhanced baryon growth
BAO scale	from sound horizon	same sound horizon
Galaxy rotation	requires CDM halo	environmental screening

The two frameworks agree on all linear observables. They differ on nonlinear scales (rotation curves, cluster profiles), where MEG’s environmental screening produces its distinctive predictions.

## 9 The Lyman- $\alpha$ Forest Test

The Lyman- $\alpha$  forest is the most demanding small-scale test of any dark matter alternative. Absorption features in quasar spectra at  $z \sim 2\text{--}5$  probe the intergalactic medium (IGM) on comoving scales  $k \sim 0.1\text{--}10 \ h \text{ Mpc}^{-1}$ . Any departure from CDM-like *shape* in the power spectrum across this band would appear as a tilt or damping in the 1D flux spectrum, and is strongly constrained by BOSS/eBOSS/DESI data.

### 9.1 Why MEG passes: the IGM is unscreened and sub-Helmholtz

The diffuse IGM at  $z \sim 3$  has mean baryon density  $\bar{\rho}_b \sim 10^{-28} \text{ kg/m}^3$ , many orders of magnitude below the screening density  $\rho_0 \sim 10^{-21} \text{ kg/m}^3$ . The vacuum entropy response is therefore completely unscreened in the IGM, and the full MEG kernel operates.

The MEG coherence length in the IGM at  $z \sim 3$  is  $\lambda_c(z=3) \simeq 0.18 h^{-1} \text{ Mpc}$  (from the quarter-power screening law with the  $a^{7/4}$  running derived in the monograph). For forest scales:

$$k\lambda_c = \{0.02, 0.09, 0.18, 0.54, 1.8\} \quad \text{for} \quad k = \{0.1, 0.5, 1, 3, 10\} h \text{ Mpc}^{-1}. \quad (23)$$

Across most forest scales ( $k \lesssim 3 h \text{ Mpc}^{-1}$ ),  $k\lambda_c \ll 1$  and the MEG boost is effectively scale-independent. Only the very highest- $k$  modes begin to feel the Yukawa falloff.

## 9.2 Three diagnostic tests

A lightweight linear pipeline was constructed in the monograph to test MEG against the forest. The pipeline uses: (i) a flat  $\Lambda\text{CDM}$  background, (ii) Eisenstein–Hu no-wiggle transfer function, (iii)  $k$ -dependent linear growth with the MEG kernel, and (iv) a standard FGPA mapping with Kaiser RSD and thermal broadening for the 1D flux power. Astrophysical nuisance parameters (IGM thermal state, UV background) are held fixed and identical in both models.

**Test A: Scale-independent growth.** Linear growth factors  $D(k, a)$  were computed for five representative modes  $k = \{0.1, 0.5, 1, 3, 10\} h \text{ Mpc}^{-1}$  at  $z = 3$ . All five curves are visually indistinguishable when normalised: no resolvable  $k$ -dependence across the forest band. MEG does not imprint spurious scale structure.

**Test B: Flat 3D power ratio.** The ratio  $P_{\text{MEG}}(k)/P_{\Lambda\text{CDM}}(k)$  at  $z = 3$  is  $\simeq 1.10 \pm 0.02$  and flat over  $k = 0.1\text{--}3 h \text{ Mpc}^{-1}$ , with only a gentle taper for  $k \gtrsim 3 h \text{ Mpc}^{-1}$  (where  $k\lambda_c \sim 1$ ). The ratio is a constant rescaling, not a shape change.

**Test C: Flat 1D flux power ratio.** After FGPA + RSD + thermal mapping, the flux ratio  $P_{\text{MEG}}^{\text{1D}}/P_{\Lambda\text{CDM}}^{\text{1D}}$  remains nearly constant at  $\sim 1.15\text{--}1.20$  across the observed  $k_{\parallel}$  band. The *barcode shape* is preserved; its contrast is uniformly higher.

## 9.3 Interpretation

**Lyman- $\alpha$  result.** In the forest window ( $k \simeq 0.1\text{--}10 h \text{ Mpc}^{-1}$  at  $z \sim 2\text{--}5$ ), MEG predicts the same shape as  $\Lambda\text{CDM}$  for both the 3D matter spectrum and the 1D flux spectrum, with a nearly scale-independent enhancement ( $\sim 10\%$  in matter power,  $\sim 15\text{--}20\%$  in flux power). This enhancement is degenerate with a single amplitude parameter (e.g.  $\sigma_8$  or bias) and is absorbed by standard nuisance calibration in observational analyses. The forest demands CDM-like *shape* on small scales. MEG produces that shape because the IGM sits in the unscreened, sub-Helmholtz regime where the entropy-sourced boost is scale-independent. There is no tuned cancellation, no hidden pressure term, and no lensing-dynamics slip.

## 9.4 Falsifiable prediction

MEG predicts a nearly scale-independent enhancement of the 1D flux power by  $\sim 15\text{--}20\%$  at  $z \sim 3$ . Any robust detection of a *tilt* in  $P^{\text{1D}}$  over  $k_{\parallel} \simeq 0.1\text{--}1 h \text{ Mpc}^{-1}$  that cannot be absorbed by standard IGM thermodynamics would challenge the MEG parameterisation. The next step is a parameter-free forward model: feed observed baryon temperature-density relations and mean flux into the same pipeline to confront BOSS/eBOSS/DESI bandpowers directly.

## 10 Observational Tests and Predictions

While MEG matches  $\Lambda$ CDM on linear scales, the framework makes specific predictions that could distinguish the two:

1. **CMB Helmholtz signature:** The Jeans scale at recombination ( $k_\mu \approx 0.08 \text{ Mpc}^{-1}$ ) produces a scale-dependent effective equation of state (P45). This affects the high- $\ell$  CMB peaks and is absent in  $\Lambda$ CDM.
2. **Transition to screening:** At the boundary of nonlinear structures, the screening transitions from  $\mu_{\text{prim}}$  to  $\mu_{\text{gal}}$ . The transition profile depends on the local density gradient and should produce characteristic signatures in weak lensing at cluster boundaries.
3. **No dark matter halos:** MEG does not predict extended dark matter halos. The enhanced gravitational coupling operates on the baryonic density directly. Observations of the radial dependence of gravitational effects around galaxies (rotation curves, satellite dynamics, weak lensing profiles) test whether the gravitational enhancement tracks the baryon distribution (MEG) or an independent halo (CDM).
4. **Void-scale behaviour:** In voids,  $\rho_b \ll \rho_0$  and the full primordial coupling operates. MEG predicts stronger gravitational effects in voids than  $\Lambda$ CDM, where CDM halos are sparse. This is testable with void lensing surveys.
5. **The  $\sigma_8$  tension:** Several observations suggest less clustering than  $\Lambda$ CDM predicts from the CMB (the  $S_8$  tension). If MEG's effective  $\sigma_8$  differs from  $\Lambda$ CDM due to the nonlinear screening transition, this could either resolve or exacerbate the tension. A quantitative calculation is needed.
6. **Lyman- $\alpha$  enhancement:** As shown in Section 9, MEG predicts a nearly scale-independent  $\sim 15\text{--}20\%$  enhancement of the 1D flux power at  $z \sim 3$ . A robust detection of a tilt rather than a constant rescaling would challenge MEG.

## 11 Open Questions

1. **Nonlinear structure formation.** The linear analysis shows MEG matches  $\Lambda$ CDM. The transition to the nonlinear regime — where screening activates and the coupling drops — requires  $N$ -body simulation with MEG's density-dependent modified gravity. The question is whether the nonlinear halo mass function, the concentration-mass relation, and the satellite dynamics are consistent with observations.
2. **The BAO scale.** The sound horizon is set at recombination and is the same in MEG and  $\Lambda$ CDM. But the subsequent evolution of the BAO feature depends on the growth history, which should be checked quantitatively.
3. **Derivation of  $\mu_{\text{eff}}(k, \rho_b)$  on intermediate scales.** The present analysis uses the two limiting cases (unscreened linear, screened galactic). The interpolation through the quasi-linear regime ( $\delta_b \sim 0.1\text{--}10$ ) requires the full density-dependent screening law applied self-consistently.
4. **Isocurvature constraints.** The derivation assumes the adiabatic condition  $\delta_\chi = \delta_b$ . Any isocurvature component in the primordial entropy perturbation would modify the growth and produce observable signatures in the CMB and large-scale structure. The magnitude of any isocurvature contribution should be bounded using Planck constraints.

5. **Full Boltzmann calculation.** A modified CLASS/CAMB implementation with MEG’s density-dependent kernel would provide precision  $C_\ell^{TT}$ ,  $P(k)$ ,  $\sigma_8$ , and  $f\sigma_8(z)$  predictions for direct comparison with Planck, DESI, and Euclid data.

## 12 Conclusion

**Summary.** MEG’s structure formation problem, as identified in the earlier exploratory analysis, does not exist. The earlier analysis was based on a temporal screening prescription that does not represent MEG’s actual physics. MEG’s screening is environmental: it depends on the local baryon density through the one-loop-derived law  $\ell_{\text{env}}(\rho_b) = \ell_0[1 + (\rho_b/\rho_0)^{1/3}]^{-1/4}$ . On linear cosmological scales, the baryon density is far below the screening threshold ( $\bar{\rho}_b/\rho_0 \lesssim 10^{-2}$  throughout the growth epoch  $z \lesssim 30$ ), and the enhanced gravitational coupling  $\mu_{\text{prim}} \approx 5.4$  persists unscreened.

The persistent baryon-coupled Poisson enhancement  $(1 + \mu_{\text{prim}})\rho_b\delta_b$  is derived (Section 4), not assumed: it follows from (i) the oscillating entropy modes having  $\bar{\rho}_\chi \propto a^{-3}$ , (ii) the adiabatic condition  $\delta_\chi = \delta_b$  (both components seeded by the same inflationary curvature perturbation), and (iii) negligible environmental screening on linear scales.

Baryons therefore grow with effective matter density  $\Omega_m^{\text{eff}} = \Omega_b(1 + \mu_{\text{prim}}) \approx 0.32$ , matching the  $\Lambda$ CDM growth factor under the adiabatic persistent-kernel condition. The gravitational potential persists at  $\sim 80\%$  of its recombination value. The Helmholtz Jeans scale moves far above all structure-formation modes during the growth epoch, eliminating Jeans suppression. The Lyman- $\alpha$  forest test is passed with a scale-independent  $\sim 10\text{--}15\%$  enhancement degenerate with standard astrophysical nuisance parameters.

The physical picture is simple: baryons fall into their own enhanced gravitational field. The vacuum entropy field modifies the gravitational coupling, not the source. Screening activates only where local density is high — inside virialised structures — which is precisely the regime where MEG’s rotation curve predictions operate. No dark matter particles. No temporal transitions. No two-fluid clustering problem.

## References

## References

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